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Slow-roll inflation with a Gauss-Bonnet correction

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We consider slow-roll inflation for a single scalar field with an arbitrary potential and an arbitrary nonminimal coupling to the Gauss-Bonnet term. By introducing a combined hierarchy of Hubble and Gauss-Bonnet flow functions, we analytically derive the power spectra of scalar and tensor perturbations. The standard consistency relation between the tensor-to-scalar ratio and the spectral index of tensor perturbations is broken. We apply this formalism to a specific model with a monomial potential and an inverse monomial Gauss-Bonnet coupling and constrain it by the 7-year Wilkinson Microwave Anisotropy Probe data. The Gauss-Bonnet term with a positive (or negative) coupling may lead to a reduction (or enhancement) of the tensor-to-scalar ratio and hence may revive the quartic potential ruled out by recent cosmological data.

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I. INTRODUCTION

Inflation in the early Universe has become the standard model for the generation of cosmological perturbations in the Universe, the seeds for large-scale structure and temperature anisotropies of the cosmic microwave background. The simplest scenario of cosmological inflation is based upon a single, minimally coupled scalar field with a flat potential. Quantum fluctuations of this inflaton field give rise to an almost scale-invariant power spectrum of isentropic perturbations (see Refs. [1, 2] for reviews).

String theory is often regarded as the leading candidate for unifying gravity with the other fundamental forces and for a quantum theory of gravity. It is known that the effective supergravity action from superstrings induces correction terms of higher order in the curvature, which may play a significant role in the early Universe. The simplest such correction is the Gauss-Bonnet (GB) term in the low-energy effective action of the heterotic string [3]. Such a term provides the possibility of avoiding the initial singularity of the Universe [4]. In the presence of an exponential potential for the modulus field, nonsingular cosmological solutions were found which begin in an asymptotically flat region, undergo superexponential inflation and end with a graceful exit to a phase with decreasing Hubble radius [5].

There are many works discussing accelerating cosmology with the GB correction in four and higher dimensions [6–9]. Recently it has been shown that the GB term might give rise to violent instabilities of tensor perturbations [10]. A model in which inflation is driven by the GB term and a higher-order kinetic energy term was studied. When the GB term dominates the dynamics of the background, tensor perturbations exhibit violent negative instabilities around a de Sitter background on small scales, in spite of the fact that scale-invariant scalar perturbations can be achieved [10]. Besides the kinetic and GB terms, a scalar potential arises naturally from supersymmetry breaking or other nonperturbative effects.

In a previous work, we investigated inflationary solutions and resulting cosmological perturbations for the special case of power-law inflation when both the GB correction and the scalar potential are present [11]. Power-law inflation happens when both the potential and the GB coupling take an exponential form. In this model instabilities of either scalar or tensor perturbations show up on small scales for GB-dominated inflation. The GB correction with a positive (or negative) coupling may lead to a reduction (or enhancement) of the tensor-to-scalar ratio in the potential-dominated case. This effect leads to tight constraints on the magnitude of the GB correction from the Wilkinson Microwave Anisotropy Probe (WMAP) 5-year analysis [12].

Here we generalize our previous work to the more general case of slow-roll inflation with an arbitrary potential and an arbitrary coupling. Making use of a combined hierarchy (ϵ_i, δ_i) of Hubble and GB flow functions (as defined below) with $|\epsilon_i| \ll 1$ and $|\delta_i| \ll 1$, analogous to the standard slow-roll approximation, we derive the power spectra of scalar and tensor perturbations. In this scenario the spectral index of scalar perturbations contains not only the Hubble flow parameters but also the GB flow parameters. Moreover, the standard consistency relation of single-field slow-roll inflation is modified. In order to impose observational constraints on such models, we focus on a specific model with a monomial potential and an inverse monomial GB coupling. We analyze the influence of the GB term on the scalar spectral index $n_{\mathcal{R}}$ and the tensor-to-scalar ratio r.

This paper is organized as follows. In Sec. II we define the Hubble and GB flow functions. Then by using the background equations of motion, we demonstrate that the slow-roll solution exists and is stable under asymptotic conditions. In Sec. III we calculate the power spec-

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tra of scalar and tensor perturbations for the slow-roll inflation. In Sec. IV our approach is applied to a specific example. Section V is devoted to conclusions.

II. SLOW-ROLL INFLATION WITH THE GB CORRECTION

We consider the following action

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{\omega}{2}(\nabla\phi)^2-V(\phi)-\frac{1}{2}\xi(\phi)R^2_{\rm GB}\right], \quad (1)$$

where ϕ is a scalar field with a potential $V(\phi)$, $\omega=\pm1$, R denotes the Ricci scalar, $R^2_{\rm GB}=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2$ is the GB term, and $\xi(\phi)$ is the GB coupling. We work in Planckian units, $\hbar=c=8\pi G=1$. In a spatially flat Friedmann-Robertson-Walker universe with scale factor a, the background equations read

$$6H^2=\omega\dot\phi^2+2V+24\dot\xi H^3, \quad (2)$$

$$2\dot H=-\omega\dot\phi^2+4\ddot\xi H^2+4\dot\xi H\left(2\dot H-H^2\right), \quad (3)$$

$$\omega\left(\ddot\phi+3H\dot\phi\right)+V_{,\phi}+12\xi_{,\phi}H^2\left(\dot H+H^2\right)=0, \quad (4)$$

where a dot represents the time derivative, $(...)_{,\phi}$ denotes a derivative with respect to ϕ, and $H\equiv\dot a/a$ denotes the expansion rate. Since the GB coupling is a function of ϕ, one has $\dot\xi=\xi_{,\phi}\dot\phi$ and $\ddot\xi=\xi_{,\phi\phi}\dot\phi^2+\xi_{,\phi}\ddot\phi$.

Besides the slow-roll conditions $\dot\phi^2\ll V$ and $|\ddot\phi|\ll 3H|\dot\phi|$, well known for minimal-coupled single-field inflation, we impose two extra conditions, namely $4|\dot\xi|H\ll1$ and $|\ddot\xi|\ll|\dot\xi|H$. The background equations are approximately given as

$$H^2\simeq\frac{1}{3}V, \quad (5)$$

$$\dot H\simeq-\frac{1}{2}\omega\dot\phi^2-2\dot\xi H^3, \quad (6)$$

$$\dot\phi\simeq-\frac{1}{3\omega H}(V_{,\phi}+12\xi_{,\phi}H^4), \quad (7)$$

which allows us to obtain the number of e-folds

$$N(\phi)\simeq\int_{\phi_{\rm end}}^{\phi}\frac{3\omega V}{3V_{,\phi}+4\xi_{,\phi}V^2}d\phi. \quad (8)$$

Following Ref. [13] we define a hierarchy of Hubble flow parameters,

$$\epsilon_1=-\frac{\dot H}{H^2},\quad \epsilon_{i+1}=\frac{d\ln|\epsilon_i|}{d\ln a},\quad i\geq1. \quad (9)$$

The expansion is accelerated as long as $\epsilon_1<1$. In the slow-roll approximation they can be related to the usual slow-roll parameters. The new degrees of freedom introduced by the GB coupling function $\xi(\phi)$ suggest to define an additional hierarchy of flow parameters in the same way by

$$\delta_1=4\dot\xi H,\quad \delta_{i+1}=\frac{d\ln|\delta_i|}{d\ln a},\quad i\geq1. \quad (10)$$

The slow-roll approximation becomes $|\epsilon_i|\ll1$ and $|\delta_i|\ll1$.

The definition of the Hubble and GB flow parameters renders significant simplification in the involved expressions. From Eqs. (2-4) we can express the kinetic term and the potential in terms of the flow parameters:

$$\omega\dot\phi^2=[2\epsilon_1-\delta_1(1+\epsilon_1-\delta_2)]H^2, \quad (11)$$

$$V=\frac{1}{2}[6-2\epsilon_1+\delta_1(-5+\epsilon_1-\delta_2)]H^2. \quad (12)$$

We see that the potential energy dominates over the kinetic energy and the GB energy. During slow roll the sign of ω is determined by the sign of $(2\epsilon_1-\delta_1)$. In the special case of $2\epsilon_1=\delta_1$, the field is frozen, which corresponds to the constant Hubble parameter. We will not consider this special case further.

It is known that slow roll is an attractor that is rapidly approached by different initial conditions [14]. Let us demonstrate that the slow-roll solution (5-7) is the attractor of the system (2-4) under the slow-roll condition. From Eqs. (3) and (4) one has

$$\begin{aligned}&2u\left(1+24\omega\xi_{,\phi}\xi_{,\phi}H^4-4\xi_{,\phi}uH\right)H_{,\phi}=\\&\quad-\omega u^2+4\xi_{,\phi\phi}u^2H^2\\&\quad-4\xi_{,\phi}H^2\left(4uH+\omega V_{,\phi}+12\omega\xi_{,\phi}H^4\right),\end{aligned} \quad (13)$$

$$\begin{aligned}&u\left(1+24\omega\xi_{,\phi}\xi_{,\phi}H^4-4\xi_{,\phi}uH\right)u_{,\phi}=\\&\quad-3uH-\omega\left(1-4\xi_{,\phi}uH\right)V_{,\phi}+6\xi_{,\phi}H^2\big(3u^2\\&\quad-4\omega\xi_{,\phi\phi}u^2H^2-2\omega H^2+12\omega\xi_{,\phi}uH^3\big),\end{aligned} \quad (14)$$

subject to the Friedmann constraint equation

$$6H^2=\omega u^2+2V+24\xi_{,\phi}uH^3, \quad (15)$$

where $u=\dot\phi$. Suppose $\bar H(\phi)$ and $\bar u(\phi)$ is the slow-roll solution to the system (13-15). Add to this a linear homogeneous perturbation $\delta H(\phi)$ and $\delta u(\phi)$; the attractor condition will be satisfied if it becomes small as the Universe expands. Inserting $H(\phi)=\bar H(\phi)+\delta H(\phi)$ and $u(\phi)=\bar u(\phi)+\delta u(\phi)$ into Eqs. (13-15), we find that the linear perturbations satisfy

$$\delta H_{,\phi}=-\frac{3H}{u}\left[1+\frac{\delta_1\epsilon_1}{2\epsilon_1-\delta_1}+\mathcal{O}(\delta_1\epsilon_1,\delta_1\delta_2)\right]\delta H, \quad (16)$$

$$\begin{aligned}\delta u_{,\phi}=&-\frac{3H}{u}\Bigg[1+\frac{2\epsilon_1\epsilon_2-8\epsilon_1\delta_1-\delta_1\delta_2-8\delta_1^2}{6(2\epsilon_1-\delta_1)}\\&+\mathcal{O}(\delta_1\epsilon_1,\delta_1\delta_2)\Bigg]\delta u,\end{aligned} \quad (17)$$

which have an approximately decaying solution with $\delta H\propto\exp(-3N)$ and $\delta u\propto\exp(-3N)$ if the Hubble and GB flow parameters vary slowly, and hence all linear perturbations die away exponentially fast as the number of e-folds increases.

III. POWER SPECTRA

At linear order in perturbation theory, the Fourier modes of curvature perturbations satisfy [15]

$$v'' + \left(c_{\mathcal{R}}^2 k^2 - \frac{z_{\mathcal{R}}''}{z_{\mathcal{R}}}\right) v = 0, \tag{18}$$

where a prime represents a derivative with respect to conformal time $\tau = \int a^{-1} dt$, and where $z_{\mathcal{R}}$ and $c_{\mathcal{R}}$ are given by

$$z_{\mathcal{R}}^2 = \frac{a^2(\omega\dot{\phi}^2 + 6\Delta\dot{\xi}H^3)}{(1-\frac{1}{2}\Delta)^2 H^2}, \tag{19}$$

$$c_{\mathcal{R}}^2 = 1 + \frac{8\Delta\dot{\xi}H\dot{H} + 2\Delta^2 H^2(\ddot{\xi} - \dot{\xi}H)}{\omega\dot{\phi}^2 + 6\Delta\dot{\xi}H^3} \tag{20}$$

with $\Delta \equiv 4\dot{\xi}H/(1-4\dot{\xi}H)$. One can express $z_{\mathcal{R}}^2$ and $c_{\mathcal{R}}^2$ in terms of the Hubble and GB flow parameters,

$$z_{\mathcal{R}}^2 = a^2 \frac{F}{(1-\frac{1}{2}\Delta)^2}, \tag{21}$$

$$c_{\mathcal{R}}^2 = 1 - \Delta^2 \frac{2\epsilon_1 + \frac{1}{2}\delta_1(1 - 5\epsilon_1 - \delta_2)}{F}, \tag{22}$$

where $\Delta = \delta_1/(1-\delta_1)$ and $F \equiv 2\epsilon_1 - \delta_1(1+\epsilon_1-\delta_2) + \frac{3}{2}\Delta\delta_1$. The effective mass term in the scalar mode equation (18) reads

$$\begin{aligned}\frac{z_{\mathcal{R}}''}{z_{\mathcal{R}}} &= a^2 H^2 \Bigg[2 - \epsilon_1 + \frac{3}{2}\frac{\dot{F}}{HF} + \frac{3}{2}\frac{\dot{\Delta}}{H(1-\frac{1}{2}\Delta)} \\ &\quad + \frac{1}{2}\frac{\ddot{F}}{H^2 F} + \frac{1}{2}\frac{\ddot{\Delta}}{H^2(1-\frac{1}{2}\Delta)} - \frac{1}{4}\frac{\dot{F}^2}{H^2F^2} \\ &\quad + \frac{1}{2}\frac{\dot{\Delta}^2}{H^2(1-\frac{1}{2}\Delta)^2} + \frac{1}{2}\frac{\dot{\Delta}}{H(1-\frac{1}{2}\Delta)}\frac{\dot{F}}{HF}\Bigg],\end{aligned} \tag{23}$$

with

$$\begin{aligned}\frac{\dot{F}}{H} &= \epsilon_1\epsilon_2(2-\delta_1) - \delta_1\delta_2(1+\epsilon_1-\delta_2-\delta_3) \\ &\quad + \frac{3}{2}\Delta\delta_2(\Delta+\delta_1), \\ \frac{\dot{\Delta}}{H} &= \Delta^2\frac{\delta_2}{\delta_1}, \\ \frac{\ddot{F}}{H^2} &= \epsilon_1\epsilon_2(-\epsilon_1+\epsilon_2+\epsilon_3)(2-\delta_1) + \epsilon_1\delta_1\delta_2(1+\epsilon_1 \\ &\quad -2\epsilon_2-\delta_2-\delta_3) - \delta_1\delta_2^2(1+\epsilon_1-\delta_2-\delta_3) \\ &\quad -\delta_1\delta_2\delta_3(1+\epsilon_1-2\delta_2-\delta_3-\delta_4) \\ &\quad + \frac{3}{2}\Delta\delta_2(\Delta+\delta_1)(-\epsilon_1+\Delta\frac{\delta_2}{\delta_1}+\delta_3) \\ &\quad + \frac{3}{2}\Delta\delta_2(\Delta^2\frac{\delta_2}{\delta_1}+\delta_1\delta_2), \\ \frac{\ddot{\Delta}}{H^2} &= \Delta^2\frac{\delta_2}{\delta_1}(-\epsilon_1+2\Delta\frac{\delta_2}{\delta_1}-\delta_2+\delta_3).\end{aligned}$$

The Fourier modes of tensor perturbations satisfy [15]

$$u'' + \left(c_T^2 k^2 - \frac{z_T''}{z_T}\right) u = 0, \tag{24}$$

where

$$z_T^2 = a^2(1-4\dot{\xi}H), \tag{25}$$

$$c_T^2 = 1 - \frac{4(\ddot{\xi}-\dot{\xi}H)}{1-4\dot{\xi}H}. \tag{26}$$

Note that the coupling ξ appears not only in the k^2 term responsible for subhorizon oscillations but also in the effective mass term z_T''/z_T. This differs from k-inflation in which the equations of motion and evolution of the tensor perturbations are not affected by nonminimal kinetic terms. In terms of the Hubble and GB flow parameters z_T^2 and c_T^2 can be written as

$$z_T^2 = a^2(1-\delta_1), \tag{27}$$

$$c_T^2 = 1 + \Delta(1-\epsilon_1-\delta_2). \tag{28}$$

The effective mass term in the tensor mode equation (24) reads

$$\begin{aligned}\frac{z_T''}{z_T} &= a^2H^2\Bigg[2-\epsilon_1-\frac{3}{2}\Delta\delta_2-\frac{1}{2}\Delta\delta_2(-\epsilon_1+\delta_2+\delta_3) \\ &\quad -\frac{1}{4}\Delta^2\delta_2^2\Bigg].\end{aligned} \tag{29}$$

If both ϵ_1 and δ_1 are constants, which corresponds to the power-law inflation with an exponential potential and an exponential GB coupling [11], then Eqs. (23) and (29) become

$$\frac{z_{\mathcal{R}}''}{z_{\mathcal{R}}} = \frac{z_T''}{z_T} = \frac{1}{\tau^2}\frac{2-\epsilon_1}{(1-\epsilon_1)^2}. \tag{30}$$

The spectral indices of scalar and tensor perturbations read exactly

$$n_{\mathcal{R}} - 1 = n_T = -\frac{2\epsilon_1}{1-\epsilon_1}, \tag{31}$$

which are consistent with the results in Ref. [11]. We note that only ϵ_1 appears in the spectral indices whether for potential-dominated or GB-dominated inflation.

In general the Hubble and GB flow parameters are functions of cosmic time. We shall assume that time derivatives of the flow parameters can be neglected during slow-roll inflation, which will allow us to obtain the leading contribution to the slow-roll approximation. Under this assumption one has $\tau^{-1} \simeq -aH(1-\epsilon_1)$ and $\tau^2 z_{\mathcal{R}}''/z_{\mathcal{R}} \equiv \nu_{\mathcal{R}}^2 - 1/4$ can be approximated to be constant. Then the general solution to Eq. (18) is a linear combination of Hankel functions

$$v = \frac{\sqrt{\pi|\tau|}}{2} e^{i(1+2\nu_{\mathcal{R}})\pi/4}\left[c_1 H_{\nu_{\mathcal{R}}}^{(1)}(c_{\mathcal{R}}k|\tau|) + c_2 H_{\nu_{\mathcal{R}}}^{(2)}(c_{\mathcal{R}}k|\tau|)\right] \tag{32}$$

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We choose $c_1 = 1$ and $c_2 = 0$, so that the usual Minkowski vacuum state is recovered in the asymptotic past ($c_{\mathcal{R}}k|\tau| \to \infty$). The power spectrum of curvature perturbations $\mathcal{P}_{\mathcal{R}} = k^3|v/z_{\mathcal{R}}|^2/2\pi^2$ on the large scales ($c_{\mathcal{R}}k \ll aH$) is

$$\begin{aligned}\mathcal{P}_{\mathcal{R}} &= \frac{c_{\mathcal{R}}^{-3}}{|F|}\frac{H^2}{4\pi^2}\left(\frac{1-\Delta/2}{aH|\tau|}\right)^2\frac{\Gamma^2(\nu_{\mathcal{R}})}{\Gamma^2(3/2)}\left(\frac{c_{\mathcal{R}}k|\tau|}{2}\right)^{3-2\nu_{\mathcal{R}}} \\ &\simeq \frac{2^{2\nu_{\mathcal{R}}-3}c_{\mathcal{R}}^{-3}}{|F|}\frac{H^2}{4\pi^2}\frac{\Gamma^2(\nu_{\mathcal{R}})}{\Gamma^2(3/2)}\Big|_{c_{\mathcal{R}}k=aH},\end{aligned} \tag{33}$$

with spectral index

$$n_{\mathcal{R}} - 1 = 3 - 2\nu_{\mathcal{R}}. \tag{34}$$

As in the case of scalar perturbations, the power spectrum of tensor perturbations $\mathcal{P}_T = 2k^3|2u/z_T|^2/2\pi^2$ is given by

$$\begin{aligned}\mathcal{P}_T &= \frac{8c_T^{-3}}{1-\delta_1}\frac{H^2}{4\pi^2}\left(\frac{1}{aH|\tau|}\right)^2\frac{\Gamma^2(\nu_T)}{\Gamma^2(3/2)}\left(\frac{c_Tk|\tau|}{2}\right)^{3-2\nu_T} \\ &\simeq 2^{2\nu_T}c_T^{-3}\frac{H^2}{4\pi^2}\frac{\Gamma^2(\nu_T)}{\Gamma^2(3/2)}\Big|_{c_Tk=aH},\end{aligned} \tag{35}$$

with spectral index

$$n_T = 3 - 2\nu_T, \tag{36}$$

where we have defined $\nu_T^2 \equiv \tau^2 z_T''/z_T + 1/4$. All background quantities above are evaluated at the moment such that $c_Tk = aH$. This is not exactly the same time as the horizon-crossing time in Eq. (33) for scalar modes, but to lowest order in the slow-roll parameters this difference is unimportant. We can use the slow-roll approximation to estimate the amount of e-folds between horizon crossing of the scalar mode and the tensor mode with a reference scale k, $\Delta N \sim \ln(c_T/c_{\mathcal{R}}) \sim \delta_1/2$. An important observational quantity is the tensor-to-scalar ratio which is defined as

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} \simeq 2^{3+2\nu_T-2\nu_{\mathcal{R}}}|F|\frac{c_{\mathcal{R}}^3}{c_T^3}\frac{\Gamma^2(\nu_T)}{\Gamma^2(\nu_{\mathcal{R}})}. \tag{37}$$

To first order in the slow-roll approximation, we have

$$c_{\mathcal{R}}^2 \simeq 1 - \frac{\delta_1^2(4\epsilon_1+\delta_1)}{2(2\epsilon_1-\delta_1)}, \tag{38}$$

$$\frac{z_{\mathcal{R}}''}{z_{\mathcal{R}}} = a^2H^2\left[2 - \epsilon_1 + \frac{3(2\epsilon_1\epsilon_2 - \delta_1\delta_2)}{2(2\epsilon_1-\delta_1)} + \mathcal{O}(\epsilon_1\epsilon_2, \delta_1\delta_2)\right], \tag{39}$$

$$c_T^2 \simeq 1 + \delta_1, \tag{40}$$

$$\frac{z_T''}{z_T} = a^2H^2\left[2 - \epsilon_1 + \mathcal{O}(\delta_1\delta_2)\right]. \tag{41}$$

The spectral indices of scalar and tensor perturbations read

$$n_{\mathcal{R}} - 1 \simeq -2\epsilon_1 - \frac{2\epsilon_1\epsilon_2 - \delta_1\delta_2}{2\epsilon_1 - \delta_1}, \tag{42}$$

$$n_T \simeq -2\epsilon_1, \tag{43}$$

which show that the spectral index of scalar perturbation contains not only the Hubble flow parameters but also the GB flow parameters. Even for a solution very close to de Sitter inflation (i.e., $\epsilon_i \approx 0$), the GB term can lead to a red ($\delta_2 > 0$) or blue ($\delta_2 < 0$) power spectrum of scalar perturbation. If $|\delta_1| \ll \epsilon_1$, the spectral indices are the same as for a potential-driven slow-roll inflation.

The tensor-to-scalar ratio (37) is approximately

$$r \simeq 8|2\epsilon_1 - \delta_1| \neq -8n_T, \tag{44}$$

which is the modified consistency relation. The degeneracy of standard consistency relation is broken in the slow-roll inflation with the Gauss-Bonnet correction. For this reason, the future experimental checking of this relation is usually regarded as an important test of the simplest forms of inflation.

The Hubble and GB flow parameters can be expressed in terms of the potential and the GB coupling

$$\epsilon_1 \simeq \frac{Q}{2}\frac{V_{,\phi}}{V}, \tag{45}$$

$$\epsilon_2 \simeq -Q\left(\frac{V_{,\phi\phi}}{V_{,\phi}} - \frac{V_{,\phi}}{V} + \frac{Q_{,\phi}}{Q}\right), \tag{46}$$

$$\delta_1 \simeq -\frac{4}{3}\xi_{,\phi}QV, \tag{47}$$

$$\delta_2 \simeq -Q\left(\frac{\xi_{,\phi\phi}}{\xi_{,\phi}} + \frac{V_{,\phi}}{V} + \frac{Q_{,\phi}}{Q}\right), \tag{48}$$

where $Q \equiv \omega(V_{,\phi}/V + 4\xi_{,\phi}V/3)$.

The key result of our paper is the general slow-roll expression for GB inflation, Eqs. (42), (43) and (44), which is new and follows from a nontrivial calculation.

IV. AN EXAMPLE MODEL

Let us consider a specific inflation model

$$V(\phi) = V_0\phi^n, \quad \xi(\phi) = \xi_0\phi^{-n}. \tag{49}$$

This potential has been widely studied. The specific choice of GB coupling allows us to find an analytic relation between the spectral index of curvature perturbations and the tensor-to-scalar ratio. If $\alpha \equiv 4V_0\xi_0/3 = 1$, all flow parameters vanish. The motion of the inflaton is frozen because the force due to the slope of the potential is exactly balanced by one, the slope of the GB coupling. In this case, exact de Sitter inflation can be realized for the monomial potential and the inverse monomial GB coupling. If $\alpha < 1$, choosing $\omega = 1$ is required for a positive ϵ_1. In this case the contribution of the positive GB term increases the Hubble expansion rate during inflation, which makes the evolution of the inflaton slower than in the case of standard slow-roll inflation, while the contribution of the negative GB term decreases the Hubble expansion rate. If $\alpha > 1$, we choose $\omega = -1$ to guarantee $\epsilon_1 > 0$. The potential force drives the inflaton to climb up the potential while the GB force drives the field

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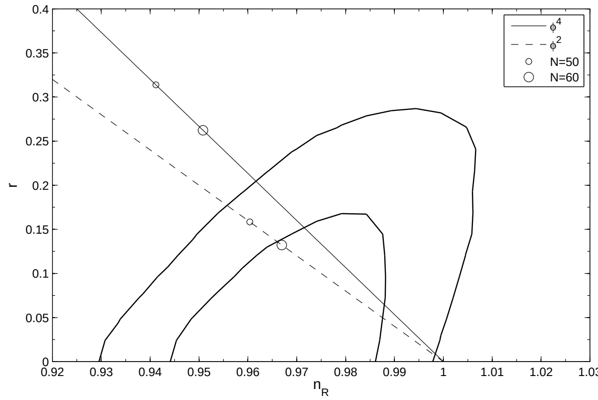

FIG. 1: Two-dimensional joint marginalized constraint (68% and 95% confidence level) on the scalar spectral index $n_{\mathcal{R}}$ and the tensor-to-scalar ratio r derived from the data combination of WMAP7+BAO+H_0 by imposing the standard consistency relation. The symbols show the predictions from the ϕ^4-potential (solid line) and ϕ^2-potential (dashed line) models with the number of e-folds equal to 50 (small) and 60 (large).

to roll down. Since the GB force dominates over the potential force, slow-roll inflation can be realized. In what follows we restrict our discussion to the case of $\alpha < 1$.

The flow parameters are

$$\epsilon_1 \simeq \frac{1}{2}n^2(1-\alpha)\phi^{-2}, \tag{50}$$

$$\epsilon_2 \simeq 2n(1-\alpha)\phi^{-2}, \tag{51}$$

$$\delta_1 \simeq n^2\alpha(1-\alpha)\phi^{-2}, \tag{52}$$

$$\delta_2 \simeq 2n(1-\alpha)\phi^{-2}. \tag{53}$$

From Eqs. (42) and (44) one gets

$$n_{\mathcal{R}} - 1 = -n(n+2)(1-\alpha)\phi^{-2}, \tag{54}$$

$$r = 8n^2(1-\alpha)^2\phi^{-2}. \tag{55}$$

Inflation ends at $\epsilon_1(\phi_{\rm end}) = 1$, which gives the value of the field at the end of inflation

$$\phi^2_{\rm end} = \frac{1}{2}n^2(1-\alpha). \tag{56}$$

Then from (8) we find the value of the field N e-folds before the end of inflation

$$\phi^2 = 2n(1-\alpha)(N + \frac{n}{4}). \tag{57}$$

The spectral index $n_{\mathcal{R}}$ and the tensor-to-scalar ratio r can be written in terms of the function of N:

$$n_{\mathcal{R}} - 1 = -\frac{2(n+2)}{4N+n}, \tag{58}$$

$$r = \frac{16n(1-\alpha)}{4N+n}. \tag{59}$$

Note that the spectral index is independent of V_0 and ξ_0, but the tensor-to-scalar ratio depends on $\alpha = 4V_0\xi_0/3$.

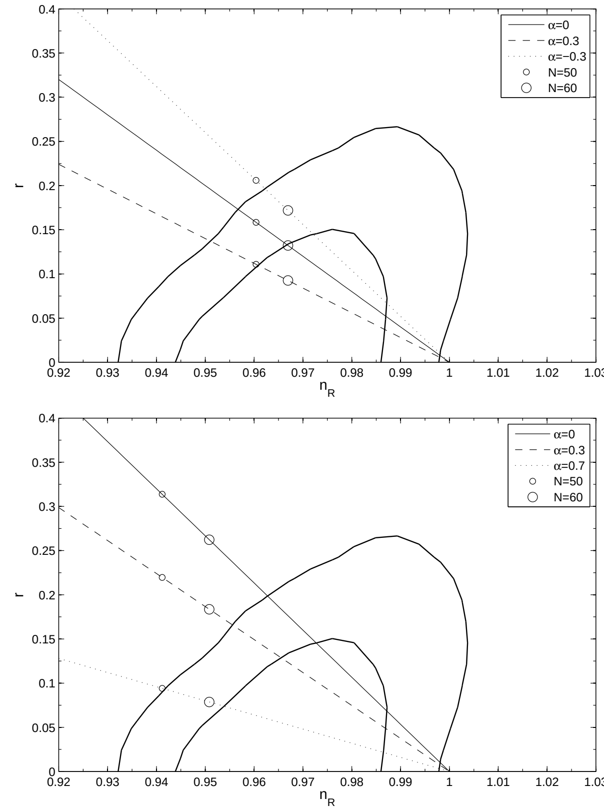

FIG. 2: Tensor-to-scalar ratio r versus the spectral index $n_{\mathcal{R}}$ for the inflation model (49) with $n = 2$ (top panel) and $n = 4$ (bottom panel). The contours show the 68% and 95% confidence level derived from WMAP7+BAO+H_0 without the consistency relation.

The GB correction leads to a reduction of the tensor-to-scalar ratio if $\xi_0 > 0$ while an enhancement if $\xi_0 < 0$, which is still valid in the power-law inflation model with the exponential potential and GB coupling [11].

Figure 1 shows the two-dimensional joint marginalized constraint (68% and 95% confidence level) on $n_{\mathcal{R}}$ and r from the 7-year WMAP+BAO+H_0 by imposing the standard consistency relation [16]. The symbols show the predictions from the ϕ^4-potential (solid line) and ϕ^2-potential (dashed line) models with the number of e-folds equal to 50 (small) and 60 (large). We can see that the predicted points with $N = 50, 60$ for the quartic potential are far away from the 95% region. The quadratic potential is consistent with the data.

However, the consistency relation $n_T = -r/8$ is broken in the slow-roll inflation with the GB correction. Therefore, in our analysis, n_T is varied independent of the tensor-to-scalar ratio. For the tensor perturbations we assume a power-law power spectrum, with a uniform prior on n_T as $-0.5 < n_T < 0$. In Fig. 2 we show the 1σ and 2σ contours derived from the data combination of WMAP7+BAO+H_0 by using the CosmoMC package [17]. Compared to the contours of Fig. 1 we find

that the joint constraint on $n_{\mathcal{R}}$ and r becomes a little tighter. The WMAP7+BAO+H_0 data do not constrain n_T. Basically all values allowed by the prior are also allowed by the potential and the coupling.

In Fig. 2 we plot the values of $n_{\mathcal{R}}$ and r in the models with $n = 2$ (top panel) and $n = 4$ (bottom panel) for different values of N and α. We can see that the model parameter α can shift the predicted r vertically for a fixed number of e-folds. For $n = 2$, the model with a positive α is more favored observationally. For $n = 4$, the model with $\alpha > 0.7$ is consistent with the data within the 95% confidence level, in which the prediction for the tensor-to-scalar ratio is smaller than the $\alpha = 0$ case while the prediction for $n_{\mathcal{R}}$ is the same as the $\alpha = 0$ case. Other ways to avoid the exclusion of the ϕ^4 potential have been studied in Ref. [18].

V. CONCLUSIONS AND DISCUSSIONS

In this paper we have studied slow-roll inflation with a nonminimally coupled Gauss-Bonnet term. We have defined a combined hierarchy (ϵ_i, δ_i) of Hubble and GB flow functions such that $|\epsilon_i| \ll 1$ and $|\delta_i| \ll 1$ is the analogue of the standard slow-roll approximation. It has been demonstrated that slow-roll solution is the attractor solution under the slow-roll condition. We have analytically derived the power spectra of scalar and tensor perturbations. In general the spectral index of scalar perturbations depends on the Hubble flow parameters and the GB flow parameters. However, the spectral index of tensor perturbations is independent of the GB flow parameters to first order in the slow-roll approximation. In this scenario the standard consistency relation does not hold because of the GB correction.

We apply our general formalism to large-field inflation with a monomial potential and the GB coupling (49). We focus on the case of $\omega = 1$ and $\alpha < 1$ since the field theory of phantom-type fields encounters the problem of stability. In this case, the GB term with the positive (or negative) coupling slows down (or speeds up) the evolution of the inflaton during inflation, which decreases (or increases) the energy scale of the potential to be in agreement with the amplitude of scalar perturbations. However the amplitude of tensor perturbations only depends on the energy scale of the potential at the horizon-crossing time. Therefore, the tensor-to-scalar ratio is suppressed for $\alpha > 0$ while it is enhanced for $\alpha < 0$.

As shown in Fig. 2, the model parameter α can shift the predicted r vertically for a fixed number of e-folds in the $n_{\mathcal{R}}$-r plane. For $n = 2$, the quadratic potential can be made a better fit to the data by the positive GB coupling. For $n = 4$, it is known that the model with $\alpha = 0$ is excluded by the WMAP7+BAO+H_0 analysis. However, in our scenario of inflation $\alpha > 0.7$ is within the 2σ contour for $N > 50$, and it is consistent with the data within the 95% confidence level.

The results of this work are generic as soon as nonminimal couplings are considered. While it is always possible by means of a conformal transformation to work in the Einstein frame and to avoid the presence of a $\phi^2 R$ term in the Lagrangian, the coupling of the scalar field to the GB term cannot be argued away by the same conformal transformation. While we studied perturbation spectra in the Einstein frame, similar properties hold in the Jordan frame.

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